Journal of Engineering Physics and Thermophysics, Vol. 69, No. 3, 1996

SPECIFIC FEATURES OF BLOOD FLOW IN A MICROCIRCULATORY CELL

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UDC 532.135:611

A mathematical model of a blood flow in a microcirculatory cell of an arbitrary morphological structure is suggested. To serve calculation of a particular case of the cell, a mathematical model of blood suspension seeping through a hemofilter is also presented on whose basis averaged elastic characteristics of erythrocyte membranes are calculated using the data of the available experiments. This is employed for diagnostics and control of the efficiency of the used methods of medical treatment.

In the study of the processes that take place in a living body, methods of mathematics and mechanics are widely employed. One of the most important objects of these studies is a microcirculatory circuit that represents the systems of blood circulation. It should be noted that it is here that the processes of tissue nutrition and slag removal take place. A microcirculatory cell, consisting of an afferent arteriole, a system of capillaries, an efferent venule and a system of shunting vessels, is the main structural unit of a microcirculatory circuit to which vessels with a diameter smaller than 100 μm are assigned. The structure and classification of vessels of a microcirculatory circuit are given in [1-4]. The problems of mathematical simulation of a blood flow in the microcirculation system are considered in [4, 5] where blood is modeled as a non-Newtonian fluid. It is noteworthy that in these works network and arcade types of a cell were not considered, whereas these very types of cells are most typical for the microcirculatory circuit of the organism.

In this work the microcirculatory cell of an arbitrary morphological structure is divided into two parts. Vessels with a diameter from 10 to 100 μm (arterioles, venules, large arterial-venular shunts) are assigned to the first part, and capillaries and intercapillary shunts with a diameter smaller than 10 μm belong to the second part. The blood flow in the first part of the microcirculatory cell is described by a model of structural continuum, which considers blood as suspension, and explicitly allows for migration of erythrocytes from a vessel wall and their pseudoturbulent diffusion [6-8]. The blood flow in capillaries is described by a structural model that considers the motion of single erythrocytes and their linear aggregates in single capillary vessels filled-in by a viscous fluid (blood plasma) and explicitly allows for deformation of erythrocytes in capillaries [9] and blood plasma seeping through a capillary wall [10], and also makes it possible to estimate the influence of nonstationary effects on blood flow in the microcirculation system [11]. According to the results of [6], if hematocrit in large blood vessels is equal to 43–45%, then a mean-over-vessel-section bulk concentration of blood cells in an arteriole is about 33% and in the main channel of the microcirculatory cell and in capillaries is less than 30%. Therefore, hydrodynamic interaction of neighboring blood cells in capillaries can be ignored and the model of blood flow (blood plasma) in a tube in the vicinity of a single erythrocyte, modeled by an elastic body or a membrane filled-in by fluid, can be used to study the blood flow in a capillary [9].

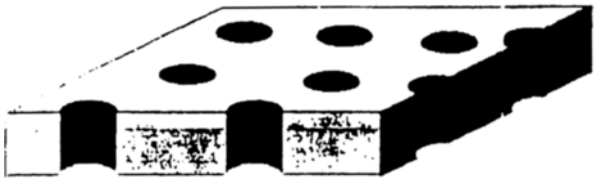
The blood flow in a microcirculatory main-line type cell is studied in [12].

The simplest version of the main-line type cell is a hemofilter (Fig. 1). A mathematical model of the process of blood cell suspension filtering through the hemofilter allows one to use the known experimental data to directly obtain averaged elastic characteristics of the erythrocyte membrane.

Let the filter porosity σ and the area of its working section S be known. Then, according to the law of resistance in parallel tubes and to the generalized Poiseuille is law, the apparent viscosity of suspension is

Taras Shevchenko National University, Kiev, Ukraine. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 69, No. 3, pp. 419-423, May-June, 1996. Original article submitted March 20, 1996.

 1062-0125/96/6903-0334$15.00



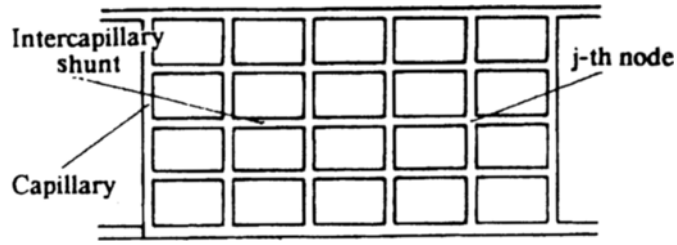


Fig. 1. A portion of the membrane of a microporous filter.

Fig. 2. Schematic representation of a microcirculatory cell of the network type.

$$\eta_{\text{eff}} = \frac{\pi R_0^4 N \Delta P t_s}{8 L_0 V_s}, \tag{1}$$

where $N = [\sigma, S]$, square brackets denote the operation of taking an integral part of the number.

Similarly, for a suspending fluid its viscosity is

$$\mu = \frac{\pi R_0^4 N \Delta t_f}{8 L_0 V_f}, \tag{2}$$

where V_f and t_f are the volume and time of suspending fluid passing through the filter under the effect of pressure drop ΔP, respectively.

The relative apparent viscosity of suspension as a whole is

$$\bar{\eta} = \frac{\eta_{\text{eff}}}{\mu} = \frac{t_s}{t_f} \frac{V_f}{V_s}. \tag{3}$$

If the suspension hematocrit is not more than 30%, the interaction of neighboring erythrocytes can be ignored. Simple recalculation allows one to pass over from the relative apparent viscosity of the suspension as a whole to the relative apparent viscosity of suspension in the vicinity of a single erythrocyte and then, using the model suggested in [9], to averaged elastic parameters of the erythrocyte membrane.

The results of calculations by the model given in [9] and experimental data of [13, 14] show that the Young modulus E for the erythrocyte membranes of healthy donors (the Poisson coefficient is assumed $\nu = 0.5$) are in good agreement with the known experimental data for single erythrocytes [15-17]. With some impairments in the organism the value of E grows greatly and can serve as an index of this impairment and also as the criterion of the efficiency of the treatment methods used.

The blood flow in microcirculatory cells of classical, bridge, arcade and network types and their combinations was also mathematically simulated (classification according to [2, 4]).

We study distribution of blood flows in the microcirculatory cell of the most general network type (Fig. 2). All geometrical dimensions of the cell are known. The blood flow occurs due to unknown pressure drops at the ends of separate vessels of which the microcirculatory cell consists. Blood will be modeled as a pseudo-Newtonian fluid with constant (or slowly varying in time) viscosity. Since the Reynolds number is small ($\text{Re} < 0.01$), the blood flow is laminar, and the curvature of vessels and nonstationary effects can be ignored.

On the basis of the law of mass conservation a total fluid flow rate in the vicinity of an arbitrary j-th node of the cell (Fig. 2) should be zero

$$\sum_i Q_i = 0, \tag{4}$$

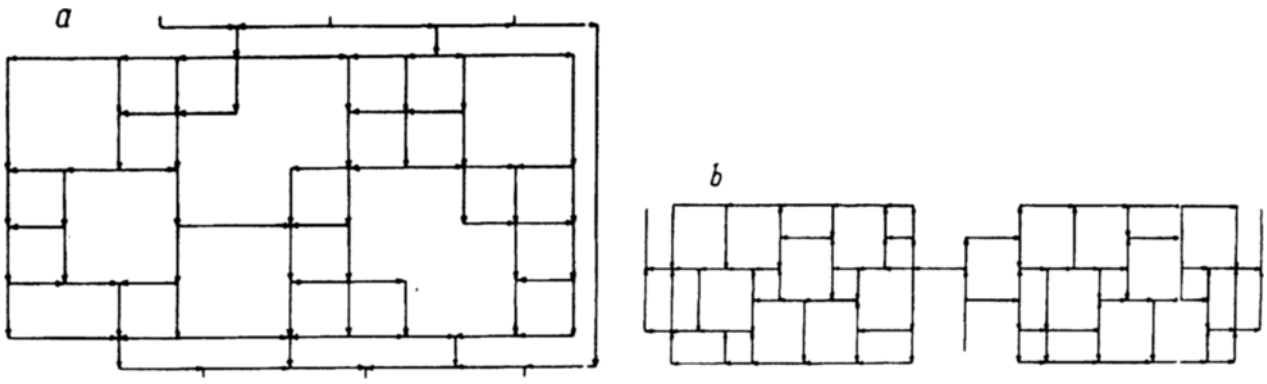


Fig. 3. Schematic distribution of blood flows in a microcirculatory cell with arcade and arcade-bridge types of the microcirculatory bed construction (a) and with a terminal type of branching of vessels (b).

where i passes all the nodes neighboring the j-th node. We use the generalized law of resistance in a separate i-th vessel of the cell

$$R_i = \frac{8\eta_{\text{eff}}^{(i)} L_i}{\pi r_i^4}. \tag{5}$$

According to the generalized Poiseuille is law

$$Q_i = \frac{\Delta P_i}{R_i}, \tag{6}$$

where

$$\Delta P_i = P_j - P_i. \tag{7}$$

Substituting (5), (6), and (7) into (4), we obtain a system of algebraic equations relative to unknown pressures P_i at the cell nodes

$$P_j \sum_i \alpha_i - \sum_i \alpha_i P_i = 0, \tag{8}$$

where j is the index of the considered node; i are the indices of neighboring nodes; $\alpha_i = 1/R_i$.

To conduct calculations and a numerical experiment, the computer program MS Windows 3.1 was created for a medium. This program is a graphical editor that allows one to interactively produce and edit the configuration of microcirculatory cells. It models cells via planar graphs where the linear parameters (length, diameter) etc. can be set for any fin (capillary, shunt). The computational part of the program constructs a system of linear equations (8), which corresponds to the chosen configuration and calculates pressure distribution in the microcirculatory cell with allowance for assigned parameters. The service tools of the program make it posible to look through the cells in an arbitrary scale, preserve configurations in files, and obtain hard copies of the calculation results.

We studied microcirculatory cells with arcade and arcade-bridge types of the microcirculatory bed construction (Fig. 3a) and with a terminal type of vessel branching (Fig. 3b) that are an elementary level of the organization of the microcirculatory bed, which provides maintenance of tissue homeostasis in separate microregions of organs. Microcirculatory cells of network and classical types of various configurations were also studied.

Thus, this paper uses structural and structural-continual models of a blood flow in a separate capillary, arteriole, and venule to propose a model that describes the blood flow in a microcirculatory cell of an arbitrary morphology.

A mathematical model of blood suspension seeping through a hemofilter is also suggested on whose basis one can calculate averaged elastic characteristics of erythrocyte membranes using the data of available experiments. This can be used for diagnostics and control of the efficiency of the used methods of treatment.

NOTATION

ΔP, pressure drop; μ, viscosity of suspending fluid; σ, porosity of filter; S, area of the filter working section; N, quantity of pores on the filter working section; R_0, pore diameter; L_0, pore length (filter thickness); V_s, t_s, volume and time of suspension passing through a filter; Q_i, fluid flow rate between the i-th nodes; R_i, resistance to blood flow in the i-th vessel; η, relative apparent viscosity of suspension; η_{eff}^i, apparent viscosity of blood in the i-th vessel; r_i, radius of the i-th vessel; L_i, length of the i-th vessel; α_i, conductivity of the i-th vessel; E, Young's modulus; ν, Poisson coefficient.

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